CHAPTER

Properties of gases

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Viscosity and other fluid properties are the material preconditions for fluid lubrication. Because of the compressibility and the strong coupling between density and temperature and pressure, lubrication gases show different viscosity−pressure and viscosity−temperature relationships compared to liquids. In addition, water held in gas often leads to a condensing problem in high-pressure gas seals, which also makes characteristics of gas lubrication different from those of liquid lubrication. Essentially, it depends on the physical properties of the gas.

This chapter introduces a basic knowledge of gases related to lubrication calculation, including equations of the gas state; the relationship between viscosity, and pressure and temperature; and the relationship between humidity and pressure and temperature.

1.1 Gas equations

In gas lubrication, the flow of gas is also the process of changing the state of gas. Generally, the gas lubrication Reynolds equation describes the macroscopic motion of the gas under external forces such as velocity shear and extrusion, and the energy equation presents the relationship between gas and external heat exchange and the transformation of macroscopic mechanical energy and gas. The description of the state of the gas microthermal motion is described by the three parameters of pressure p, density ρ, and temperature T. For an ideal gas, the relationship between pressure, density, and temperature satisfies the ideal gas state equation,

$$\frac{p}{\rho} = R_{\mathrm{u}} T \tag{1.1}$$

where the ideal gas constant $R_{\mathrm{u}} = 8.314472\ \mathrm{m^3\ Pa/(mol\ K)}$.

For a general thermal process when the pressure, density, and temperature change at the same time, a single use of the ideal gas state equation cannot give change values of the pressure, seal, and temperature because there are three variables in the equation. So, we need to build another gas equation.

The Brownian motion, discovered in 1827 by British botanist Gordn Brown, is a state of movement of microscopic particles in gases or liquids, and was discovered in 1827 by British botanist Gordon Brown. In 1907, Einstein proposed the energy equipartition principle. This basic theory of statistical mechanics holds that the kinetic energy of a microscopic particle depends only on its temperature, regardless of its size or mass. However, it had been unable to directly prove the equipartition theorem for Brownian particles because the high-speed collision between the particles in Brownian motion led to constant direction and speed change, making the instantaneous velocity of a particle in Brownian motion difficult to measure. In 2010, Li et al.'s experimental work [1] proved that the energy equipartition principle is correct for air. This provides a new way to discuss gas thermal effect in gas lubrication.

We discussed and analyzed the characterization of gas pressure and temperature based on the energy equipartition principle, and established the independent equation of pressure and temperature for an ideal gas, which makes it possible to calculate the gas temperature field for any thermal process [2]. The gas equations based on the energy equipartition principle are introduced in this section.

1.1.1 Ideal gas equations

Generally, ideal gas molecule movement can be dealt with using the rigid sphere model. Fig. 1.1 illustrates freedom of an ideal gas molecular motion. As shown in Fig. 1.1, an ideal gas molecular motion includes translation and rotation motions in the x, y, and z directions. When temperature is high, there are additional vibration motions for gas molecular movement. Motion in each of these directions is counted as one degree of freedom. According to the energy equipartition principle, the energy per degree of freedom for these kinds of motions is equal to E_{m}. So, gas molecular energy can be expressed as follows:

$$E_{\mathrm{sum}} = i_{\mathrm{d}}\overline{E}_{\mathrm{e}} \tag{1.2}$$

where i_{d} is the degree of gas molecular freedom.

Now, we assume that gas temperature only tokens macroscopically inner energy of the gas molecular, so the following equation can be given:

$$T = c_{\mathrm{T}} i_{\mathrm{d}}\overline{E}_{\mathrm{e}} \tag{1.3}$$

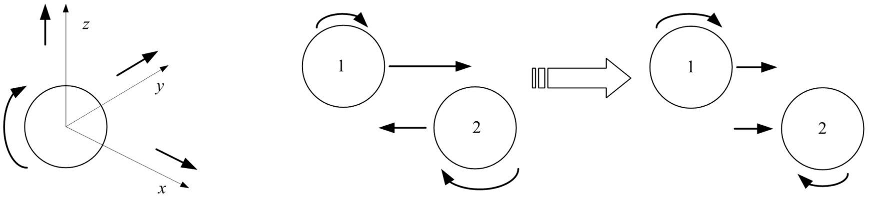

FIGURE 1.1

Degree of gas molecular freedom.

where T is the absolute temperature and c_T is the temperature coefficient. c_T^{-1} means the increase of gas molecular energy per unit temperature increment, that is, the reciprocal of specific heat at constant volume. So the following equation can be given:

$$c_T = \frac{1}{c_v} \tag{1.4}$$

where c_v is specific heat at constant volume.

Furthermore, it is assumed that gas pressure is determined only by both translation energy of gas molecular energy and gas density. Hence,

$$p = c_p \rho i_d \overline{E}_e \tag{1.5}$$

where c_p is the pressure coefficient.

From Eqs. (1.3) and (1.5), the ideal gas state equation can be obtained as follows:

$$\frac{p}{T\rho} = \frac{c_p}{c_T} \tag{1.6}$$

So, it can be obtained by comparing it with Eq. (1.1):

$$c_p = c_T R_u \tag{1.7}$$

That is,

$$c_p = \frac{R_u}{c_v} \tag{1.8}$$

It should be noted that Eqs. (1.3) and (1.5) are two independent equations that agree well with the traditional ideal gas state equation and characterize the relationship between gas pressure and temperature, and gas kinetic energy, respectively. Relative to the ideal gas equation (1.1), a gas equation has been added to realize the sealing of multivariable gas lubrication equations. In this way, the pressure, temperature, and density of gases in general thermal processes can be quantitatively analyzed.

1.1.2 Gas index equation

The gas index equation is often used to characterize and describe the thermal process of gases. Here, the index equation is derived from the energy equipartition principle, to illustrate the proposed temperature equation and pressure equation.

Let us consider 1 mol gas with the initial state of T_0, p_0, ρ_0 in the adiabatic compression process, as shown in Fig. 1.2. Assuming the sectional area is 1, the work done by the external force on the gas can be expressed as

$$\begin{aligned} \Delta w &= -pdx \\ &= -pd\left(\frac{1}{\rho}\right) \end{aligned} \tag{1.9}$$

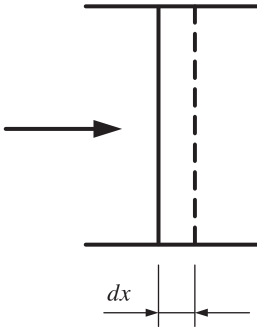

FIGURE 1.2

Illustration of gas adiabatic compression process.

According to Eq. (1.2), increase of the gas internal energy is obtained as follows:

$$\Delta w' = i_d d\overline{E}_e \tag{1.10}$$

Substituting Eq. (1.5) into the above equation gives

$$\Delta w' = \frac{1}{c_p} d\left(\frac{p}{\rho}\right) \tag{1.11}$$

For the adiabatic compression process, there is

$$\Delta w = \Delta w' \tag{1.12}$$

Hence,

$$-pd\left(\frac{1}{\rho}\right) = \frac{1}{c_p} d\left(\frac{p}{\rho}\right) \tag{1.13}$$

Integrating Eq. (1.13) gives the following expression:

$$-(\ln \rho^{-1} - \ln \rho_0^{-1}) = \frac{1}{c_p}(\ln(p\rho^{-1}) - \ln(p_0\rho_0^{-1})) \tag{1.14}$$

Furthermore, the following expression for the well-known gas index equation can be obtained from Eq. (1.14).

$$p\rho^{-\gamma} = p_0\rho_0^{-\gamma} \tag{1.15}$$

where $\gamma = 1 + c_p = 1 + R_u/c_v$

Obviously, Eq. (1.15) indicates that the gas state presents exponential change during the adiabatic compression. For air, the specific heat at constant volume equals $c_v = 2.5R_u$, so we get $\gamma = 1.4$. For the general gas thermal process, the changes of gas pressure, density, and temperature are subject to Eqs. (1.3) and

(1.5). Based on the initial sate (T_0,p_0,ρ_0) and the final state (T_1,p_1,ρ_1), the index γ can be calculated by the following expression:

$$\gamma = \frac{\ln(p_1/p_0)}{\ln(\rho_1/\rho_0)} \tag{1.16}$$

It should be noted that the gas index equation is not suitable for analysis of the gas thermal hydrodynamic lubrication problem, as the process index of the general thermal process is obtained mainly through experimental measurement.

1.1.3 Actual gas equation

For an actual gas, the relationship between pressure, density, and temperature satisfies the actual gas state equation:

$$\frac{p}{\rho} = \varepsilon R_{\mathrm{u}} T \tag{1.17}$$

where ε is gas compressibility. The gas compressibility is mainly characterized by the compressibility of the gas, so the pressure equation for actual gas is obtained as follows by revising Eq. (1.5).

$$p = \varepsilon c_{\mathrm{p}} \rho i_{\mathrm{d}} \overline{E}_{\mathrm{e}} \tag{1.18}$$

Table 1.1 shows the gas compressibility of nitrogen under different temperatures and pressures. It can be seen that in a standard room temperature under 10 MPa pressure, the variation of the gas compressibility is less than 1%. Therefore the ideal gas hypothesis is reasonable in general gas lubrication analysis.

1.1.4 Degree of gas molecular freedom

A gas molecule at room temperature can be considered as a rigid body without considering the freedom of vibration. For a nonrigid molecule, there is also a spring-like vibration, so we add 1 to the degree of freedom.

Monatomic molecules, such as helium, neon, and argon, have only one atom, which can be seen as a free particle, so there are three translational degrees of freedom.

In rigid diatomic molecules, such as hydrogen, oxygen, and carbon monoxide, the distance between the two atoms remains the same, and to determine the location of the center of mass, three independent coordinates are required. To determine the location of the point line, two independent coordinates are needed. It makes no sense to rotate the two points around the wire. Therefore the rigid diatomic molecule has five degrees of freedom: three translational degrees of freedom and two rotational degrees of freedom.

Table 1.1 Gas compressibility of nitrogen.

Pressure (MPa)	Temperature (K)								
	126.9	**143.2**	**173.2**	**203.2**	**223.2**	**248.2**	**273.2**	**293.2**	**323.2**
0.101	–	0.9939	0.9968	0.9985	0.9991	0.9991	1.0000	0.9999	1.0001
1.013	–	0.9298	0.9637	0.9805	0.9866	0.9918	0.9962	0.9975	1.0005
2.027	0.7620	0.8520	0.9266	0.9587	0.9732	0.9841	0.9925	0.9961	1.0010
3.040	0.5749	0.7642	0.8892	0.9425	0.9610	0.9781	0.9894	0.9948	1.0017
4.053	–	0.6654	0.8525	0.9210	0.9495	0.9720	0.9870	0.9940	1.0030
5.066	–	0.5616	0.8172	0.9030	0.9391	0.9676	0.9848	0.9941	1.0045
6.080	–	0.4738	0.7840	0.8901	0.9298	0.9647	0.9840	0.9942	1.0065
8.106	–	0.5698	0.7307	0.8648	0.9151	0.9576	0.9835	0.9958	1.0064
10.133	–	–	0.7053	0.8554	0.9087	0.9550	0.9848	1.0016	1.0182
20.265	–	–	–	0.9174	0.9614	1.0073	1.0355	1.0537	1.0770
30.398	–	–	–	1.0828	1.0999	1.1204	1.1335	1.1454	1.1590

Table 1.2 Degree of gas molecular freedom.

Molecular type		**Degree of freedom**			
		Translation i_t	**Rotation** i_r	**Vibration** i_s	$i_d = i_t + i_r + i_s$
Monatomic molecule		3	0	0	3
Diatomic molecule	Rigid	3	2	0	5
	Nonrigid	3	2	1	6
Polyatomic molecule	Rigid	3	3	0	6
	Nonrigid	3	3	$3n - 6$	$3n$

Table 1.3 Composition of standard dry air.

Composition	**Relative molecular mass**	**Mole fraction**	**Freedom number of gas motions**	**Constant volume specific heat capacity J/(g K)**
O_2	32.000	0.2095	5	0.649
N_2	28.016	0.7809	5	0.741
Ar	39.944	0.0093	3	–
CO_2	44.010	0.0003	6	–
H_2	–	–	–	9.934
H_2O	–	–	–	1.381

For rigid triatoms or polyatomic molecules, such as water, carbon dioxide, and ammonia, as long as the atoms are not aligned in a straight line, they can be seen as free rigid bodies with six degrees of freedom.

The degree of gas molecular freedom is shown in Table 1.2.

1.1.5 Specific heat capacity

For 1 mol of gas, when the volume remains constant, in the absence of chemical reaction and phase change, the heat absorbed or released by 1 K temperature change is called the constant volume molar heat capacity of the gas.

Table 1.3 shows the composition of standard dry air and its constant volume specific heat capacity.

1.2 Viscosity

In the theory of fluid lubrication, dynamic viscosity is used to characterize the movement resistance of a fluid, which is decisive for the load capacity of

Table 1.4 Dynamic viscosity of the air under different temperatures.

Temperature	Dynamic viscosity	Temperature	Dynamic viscosity	Temperature	Dynamic viscosity
T (°C)	**η ($\times 10^{-6}$ Pa s)**	**T (°C)**	**η ($\times 10^{-6}$ Pa s)**	**T (°C)**	**η ($\times 10^{-6}$ Pa s)**
5	17.340	15	17.840	25	18.340
5.5	17.365	15.5	17.865	25.5	18.365
6	17.390	16	17.890	26	18.390
6.5	17.415	16.5	17.915	26.5	18.420
7	17.440	17	17.940	27	18.450
7.5	17.465	17.5	17.965	27.5	18.470
8	17.490	18	17.990	28	18.490
8.5	17.515	18.5	18.015	28.5	18.515
9	17.540	19	18.040	29	18.540
9.5	17.565	19.5	18.065	29.5	18.565
10	17.590	20	18.090	30	18.590
10.5	17.615	20.5	18.115	30.5	18.615
11	17.640	21	18.140	31	18.640
11.5	17.665	21.5	18.165	31.5	18.665
12	17.690	22	18.190	32	18.690
12.5	17.715	22.5	18.215	32.5	18.715
13	17.740	23	18.240	33	18.740
13.5	17.765	23.5	18.265	33.5	18.765
14	17.790	24	18.290	34	18.790
14.5	17.815	24.5	18.318	34.5	18.815

hydrodynamic lubrication film. Because of the compressibility, the viscosity of gases show different characteristics compared with those of liquids, which are obviously affected by temperature and pressure.

Table 1.4 shows the air dynamic viscosity values of one standard atmospheric pressure under the fluctuation range of room temperature. As can been from the table, dynamic viscosity increases with increase in temperature. The dynamic viscosity increases about 9% when the temperature increases 30°C under the condition of one standard atmospheric pressure.

The viscosity of gas is affected not only by temperature but also by pressure. Both temperature and pressure have a significant influence on the dynamic viscosity of a gas. For air, a temperature increase of 150°C may make dynamic viscosity increase of about 40%. In addition, dynamic viscosity may increase about 55% when pressure increases from 0.1 to 30 MPa at a temperature of 30°C [3]. However, as temperature increases, the dynamic viscosity of water vapor is opposite to that of air; that is, the dynamic viscosity of temperature decreases rapidly with an increase in temperature.

Although a chart can give an accurate value of dynamic viscosity, the mathematical expression of gas dynamic viscosity is necessary for conveniently calculating fluid lubrication.

The main calculation formulas of gas dynamic viscosity are discussed next.

1. *Satland formula*

   The formula was established for situations in which the temperature is below 2000 K, which is expressed as follows:

   $$\frac{\eta}{\eta_0} = \frac{(T/T_0)^{3/2}(T_0 + B)}{T + B} \tag{1.19}$$

   where B is the constant associated with the gas, $B = 110.4$ K for air, and η_0 is viscosity at temperature T_0.
2. *Exponential formula*

   $$\frac{\eta}{\eta_0} = \left(\frac{T}{T_0}\right)^n \tag{1.20}$$

   where the value of exponential n varies with type and temperature of gas. In the range of $90\ \mathrm{K} < T < 300\ \mathrm{K}$, $n = 8/\rho$ is desirable.
3. *The Champman–Enskog formula*

   From the Champman–Enskog equation [4], a relationship between viscosity and temperature of a gas can be obtained as follows:

   $$\eta = \frac{5}{16}\frac{\sqrt{\pi m_{\mathrm{gas}} k_{\mathrm{B}} T}}{\pi d_{\mathrm{gas}}^2} \tag{1.21}$$

   where k_{B} is the Boltzmann constant, m_{gas} is the mean molar mass, and d_{gas} is the molecular diameter.
4. *Lucas formula*

   In general, the temperature and pressure of air are measured by the critical temperature T_{c} and the critical pressure p_{c} to replace the absolute value of air based on the principle of the contrast state.

The contrast temperature of air is defined as follows:

$$T_{\mathrm{r}} = \frac{T}{T_{\mathrm{c}}} \tag{1.22}$$

The contrast pressure of air is defined as follows:

$$p_{\mathrm{r}} = \frac{p}{p_{\mathrm{c}}} \tag{1.23}$$

In 1980, Lucas [5] proposed a method of gas viscosity calculation, which is expressed as follows:

$$\eta_{\mathrm{sc}}\xi = 0.807T_{\mathrm{r}}^{0.618} - 0.357\exp(-0.449T_{\mathrm{r}}) + 0.340\exp(-4.058T_{\mathrm{r}}) + 0.018 \tag{1.24}$$

$$\xi = 37{,}916\left(\frac{T_{\mathrm{c}}}{M^3 p_{\mathrm{c}}^4}\right)^{1/6} \tag{1.25}$$

where η_{sc} is the viscosity of dry air under the standard condition (15°C, 101,325 Pa), for which the unit is cP. ξ is the correlation coefficient of viscosity, for which the unit is cP^{-1}.

If $1 < T_r < 40$, $0 < p_r \le 100$, then there is

$$\frac{\eta}{\eta_{sc}} = 1 + \frac{A_1 p_r^{1.3088}}{A_2 p_r^{A_5} + \left(1 + A_3 p_r^{A_4}\right)^{-1}} \tag{1.26}$$

where

$$A_1 = \frac{\left(1.245 \times 10^{-3}\right)\exp\left(5.1726 T_r^{-0.3286}\right)}{T_r} \tag{1.27a}$$

$$A_2 = A_1(1.6553 T_r - 1.2723) \tag{1.27b}$$

$$A_3 = \frac{0.4489 \exp\left(3.0578 T_r^{-37.7332}\right)}{T_r} \tag{1.27c}$$

$$A_4 = \frac{1.7368 \exp\left(2.2310 T_r^{-7.6351}\right)}{T_r} \tag{1.27d}$$

$$A_5 = 0.9425 \exp\left(-0.1853 T_r^{0.4489}\right) \tag{1.27e}$$

where η is the gas viscosity to be solved, for which the unit is cP.

If $T_r \le 1.0$, $P_r < (P_{vp}/P_c)$, then there is

$$\eta\xi = 0.600 + 0.760 P_r^{B_1} + \left(6.990 P_r^{B_2} - 0.6\right)(1 - T_r) \tag{1.28}$$

where

$$B_1 = 3.262 + 14.98 P_r^{5.508} \tag{1.29a}$$

$$B_2 = 1.390 + 5.746 P_r \tag{1.29b}$$

The viscosity of gas in gas seals was analyzed by Thomas et al. [6], and the results showed that the viscosity of the ideal gas was significantly different from that of the actual gas. As shown in Fig. 1.3, in a large range of pressure and temperature variation, the Lucas formula is consistent with the measured gas viscosity.

It should be noted that the influence of pressure is not considered in the Satland formula, the Exponential formula, or the Champman—Enskog formula, which does not lead to an obvious deviation analysis in the design of ordinary gas bearing lubrication. However, for high-pressure gas seals where the gas pressure may reach up to 50 MPa, the influence of pressure on viscosity cannot be ignored in analysis of lubrication.

If the gas phase mixture is chosen as the lubricating medium, the calculation formula of the critical property parameters of the mixture is as follows [5].

Critical temperature of the mixture, T_c, is expressed as

$$T_c = \sum_{i=1}^{N} y_i T_{ci} \tag{1.30}$$