1 INTRODUCTION

1.1 Importance and objectives of inventory control

For more or less all organizations in any sector of the economy, *Supply Chain Management*, i.e., the control of the material flow from suppliers of raw material to final customers, is a crucial problem. The strategic importance of this area is today fully recognized by top management. The total investment in inventories is enormous, and the control of capital tied up in raw material, work-in-progress, and finished goods offers a very important potential for improvement. Scientific methods for inventory control can give a significant competitive advantage. This book deals with a wide range of different inventory models that can be used when developing inventory control systems.

Advances in information technology have drastically changed the possibilities to apply efficient inventory control techniques. Furthermore, the recent progress in research has resulted in new and more general methods that can reduce the supply chain costs substantially. The field of inventory control has indeed changed during the last decades. It used to mean application of simple decision rules, which essentially could be carried out manually. Modern inventory control is based on quite advanced and complex decision models, which may require considerable computational efforts.

Inventories cannot be decoupled from other functions, for example purchasing, production, and marketing. As a matter of fact, the objective of inventory control is often to balance conflicting goals. One goal is, of course, to keep stock levels down to make cash available for other purposes. The purchasing manager may wish to order large batches to get volume discounts. The production manager similarly wants long production runs to avoid time-consuming setups. He also prefers to have a large raw material inventory to avoid stops in production due to missing materials. The marketing manager would like to have a high stock of finished goods to be able to provide customers a high service level.

It is seldom trivial to find the best balance between such goals, and that is why we need inventory models. In most situations some stocks are required. The two main reasons are *economies of scale* and *uncertainties*. Economies of scale mean that we need to order in *batches*. Uncertainties in supply and demand together with lead-times in production and transportation inevitably create a need for *safety stocks*. Still, most organizations can reduce their inventories without increasing other costs by using more efficient inventory control tools.

There are important inventory control problems in all supply chains. For those who are working with logistics and supply chains, it is difficult to think of any qualification that is more essential than a thorough understanding of basic inventory models.

1.2 Overview and purpose of the book

The main purpose of this book is that it should be useful as a course textbook. The structure of the book is illustrated in Figure 1.1.

After this introduction we consider different *forecasting techniques* in Chapter 2. We focus on methods like exponential smoothing and moving average procedures for estimating the future demand from historical demand data. We also provide techniques for evaluating the size of forecast errors.

Chapters 3 - 6 deal with basic inventory problems for a *single installation* and *items that can be handled independently*. More precisely, Chapter 3 presents various basic concepts. Chapter 4 deals with *deterministic lot sizing* and Chapter 5 with *safety stocks* and *reorder points*. In Chapter 6 we discuss *integration* and *optimality*.

The contents in Chapters 2 - 6 provide the foundation for an efficient standard inventory control system, which can include:

- A forecasting module, which periodically updates demand forecasts and evaluates forecast errors.

- A module for determination of reorder points and order quantities.

- Continuous or periodic monitoring of inventory levels and outstanding orders. Triggering of suggested orders when reaching the reorder points.

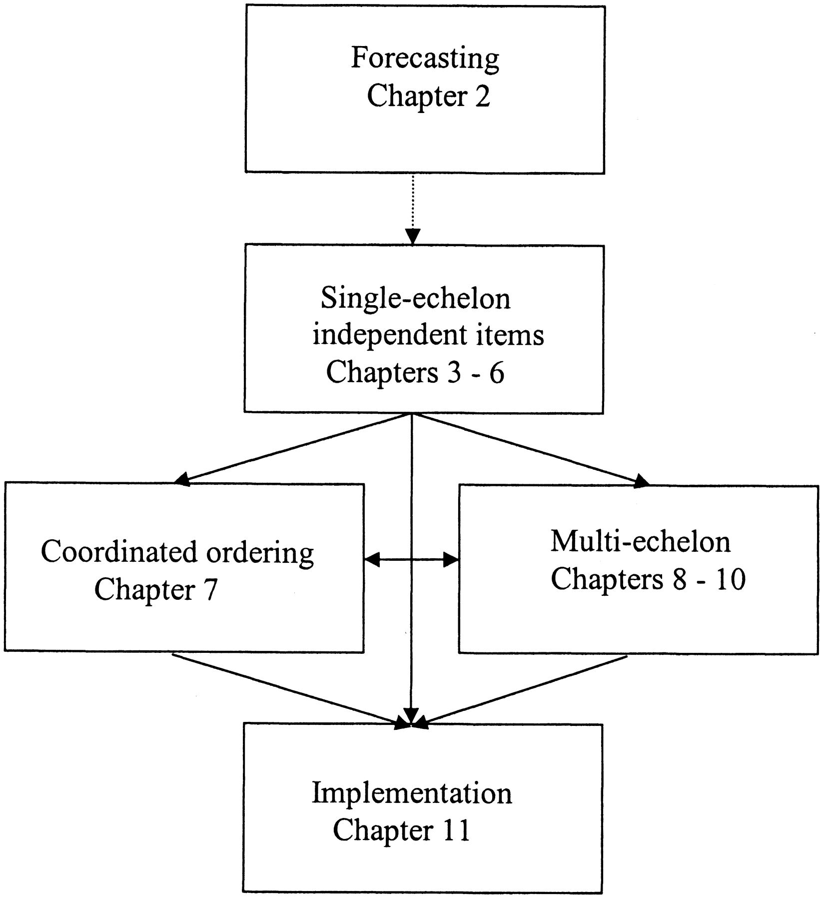

Figure 1.1 Structure of the book.

In Chapter 7 we leave the assumption of independent items and consider *coordinated replenishments*. Both production smoothing models and so-called joint replenishment problems are analyzed.

Chapters 8 - 10 focus on multi-echelon inventory systems, i.e., on several installations which are coupled to each other. The installations can represent, for example, stocks of raw materials, components, work-in-process, and final products in a production system, or a central warehouse and a number of retailers in a distribution system. In Chapter 8 we consider *structures* and *ordering policies*. Chapter 9 deals with *lot sizing* and Chapter 10 with *safety stocks* and *reorder points*.

Finally, in Chapter 11 we discuss various practical problems in connection with *implementation* of inventory control systems.

Over the years a substantial number of excellent books and overview papers dealing with various inventory control topics have been published. A selection of these publications is listed at the end of this chapter. A natural question then is why this book is needed. To explain this, note first that this book is different from most other books because it also covers very recent advances in inventory theory, for example new techniques for multi-echelon inventory systems and Roundy's 98 percent approximation. Furthermore, this book is also different from most other books because it assumes a reader with a good basic knowledge of mathematics and probability theory. This makes it possible to present different inventory models in a compact and hopefully more efficient way. The book attempts to explain fundamental ideas in inventory modeling in a simple but still rigorous way. However, to simplify, several models are less general than they could have been.

Because the book assumes a good basic knowledge of mathematics and probability theory, it is most suitable for industrial engineering and management science/operations research students. It can be used in a basic undergraduate course, and/or in a more advanced graduate course.

Chapter 2 may be omitted in a course which is strictly focused on inventory control. If it is included, it should probably be the first part of the course. Chapters 3 - 6 should precede Chapters 7 - 10. Chapter 7 can either precede or succeed Chapters 8 - 10. Chapter 11 should come at the end of the course.

An *undergraduate course* can, for example, be based on the following parts of the book: Sections 2.1 - 2.6, Sections 2.10 - 2.12, Chapters 3 - 4, Section 5.1.1, Section 5.2.1, Sections 5.3 - 5.8, Section 5.13, Section 6.3, Section 7.2.1, Section 8.1, Sections 8.2.1 - 8.2.2, Sections 8.2.4 - 8.2.5, Section 9.1, Section 9.2.1, Chapter 11.

For students that have taken the suggested undergraduate course, or a corresponding course, a *graduate course* can build on a selection of the remaining parts of the book, e.g., Sections 5.1.2 - 5.1.5, Section 5.2.2. Sections 5.9 - 5.12, Sections 5.14 - 5.15, Sections 6.1 - 6.2, Section 7.1, Section 7.3, Section 8.2.3, Sections 9.2 - 9.3, Chapter 10.

A graduate course for students that have no prior knowledge of inventory control but a good mathematical background should include most of the material suggested for the undergraduate course, but can exclude some of the sections suggested for the graduate course.

Another purpose of this book is to describe and explain efficient inventory control techniques for practitioners, and in that way simplify and promote implementation in practice. The book can, e.g., be used as a *handbook* when implementing and adjusting inventory control systems.

1.3 Framework

Models and methods in this book are based on the cost structure that is most common in industrial applications. We consider holding costs including opportunity costs of alternative investments, ordering or setup costs, and shortage costs or service level constraints. We will not deal with, for example, inventory problems related to financial speculation, i.e., when the value of an item can be expected to increase, or with aggregate planning models for smoothing production in case of seasonal demand variations. The interaction with production is recognized through setup costs but also in some models by explicit capacity constraints. The book does not cover production planning settings that are not directly related to inventory control.

The models considered in the book assume that the basic conditions for inventory control are given, for example in the form of demand distributions, lead-times, service requirements, and holding and ordering costs. In practice, most of these conditions can be changed at least in the long run. There are, consequently, many important questions concerning inventories that are related to the structure and organization of the inventory control system. Such questions may concern evaluation of investments to reduce setup costs, or whether the customers should be served through a single-stage or a multi-stage inventory system. Although we do not treat such questions directly, it is important to note that a correct evaluation must always be based on inventory models of the type considered in this book. The question is always whether the savings in inventory-related costs are larger than the costs for changing the structure of the system.

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2 FORECASTING

There are two main reasons why an inventory control system needs to order items some time before customers demand them. First, there is nearly always a *lead-time* between the ordering time and the delivery time. Second, due to certain ordering costs, it is often necessary to order in *batches* instead of unit for unit. This means that we need to look ahead and forecast the future demand. A demand forecast is an estimated average of the demand size over some future period. But it is not enough to estimate the average demand. We also need to determine how uncertain the forecast is. If the forecast is more uncertain, a larger safety stock is required. Consequently, it is also necessary to estimate the forecast error, which may be represented by the standard deviation or the Mean Absolute Deviation (*MAD*).

2.1 Objectives and approaches

In this chapter we shall consider forecasting methods that are suitable in connection with inventory control. Typical for such forecasts is that they concern a relatively short time horizon. Very seldom is it necessary to look more than one year ahead. In general, there are then two types of approaches that may be of interest:

- Extrapolation of historical data

When extrapolating historical data, the forecast is based on previous demand data. The available techniques are grounded in statistical methods for analysis of time series. Such techniques are easy to apply and use in computerized inventory control systems. It is no problem to regularly update forecasts for thousands of items, which is a common requirement in connection with practical inventory control. Extrapolation of historical data is the most common and important approach to obtain forecasts over a short horizon, and we shall devote the main part of this chapter to such techniques.

- Forecasts based on other factors

It is very common that the demand for an item depends on the demand for some other items. Consider, for example, an item that is used exclusively as a component when assembling some final products. It is then often natural to first forecast the demand for these final products, for example by extrapolation of historical data. Next we determine a production plan for the products. The demand for the considered component is then obtained directly from the production plan. This technique to "forecast" demand for dependent items is used in *Material Requirements Planning* (*MRP*) that is dealt with in Section 8.2.4.

But there are also other factors that might be reasonable to consider when forecasting demand. Assume, for example, that a sales campaign is just about to start or that a competing product is introduced on the market. Clearly this can mean that historical data are no longer representative when looking ahead. It is normally difficult to take such factors into account in the forecasting module of a computerized inventory control system. It is therefore usually most practical to adjust the forecast manually in case of such special events.

It is also possible, at least in principle, to use other types of dependencies. A forecast for the demand of ice cream can be based on the weather forecast. Consider, as another example, forecasting of the demand for a spare part that is used as a component in certain machines. The demand for the spare part can be expected to increase when the machines containing the part as a component are getting old. It is therefore reasonable to look for dependencies between the demand for the spare part and previous sales of the machines. As another example we can assume that the demand during a certain month will increase with the advertising expenditure the previous month. Such dependencies could be determined from historical data by regression analysis. (See Section 2.7.) Applications of such techniques are, however, very limited.

2.2 Demand models

Extrapolation of historical data is, as mentioned, the most common approach when forecasting demand in connection with inventory control. To determine a suitable technique, we need to have some idea of how to model the stochastic demand. In principle, we should try to determine the model from analysis of historical data. In practice this is very seldom done. With many thousands of items, this initial work does not seem to be worth the effort in many situations. In other situations there are not enough historical data. A model for the demand structure is instead determined intuitively. In general, the assumptions are very simple.

2.2.1 Constant model

The simplest possible model means that the demands in different periods are represented by independent random deviations from an average that is assumed to be relatively stable over time compared to the random deviations. Let us introduce the notation:

x_t = demand in period t,
a = average demand per period (assumed to vary slowly),
ε_t = independent random deviation with mean zero.

A constant model means that we assume that the demand in period t can be represented as

$$x_t = a + \varepsilon_t . \tag{2.1}$$

Many products can be represented well by a constant model, especially products that are in a mature stage of a product life cycle and are used regularly. Examples are consumer products like toothpaste, many standard tools, and various spare parts. In fact, if we do not expect a trend or a seasonal pattern, it is in most cases reasonable to assume a constant model.

2.2.2 Trend model

If the demand can be assumed to increase or decrease systematically, it is possible to extend the model by also considering a linear trend. Let

a = average demand in period 0,
b = trend, that is the systematic increase or decrease per period (assumed to vary slowly).

A trend model means that the demand is modeled as: